

# From solar proton burning to pionic deuterium through the Nambu–Jona-Lasinio model of light nuclei

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**Abstract.** Within the Nuclear Nambu–Jona-Lasinio model of light nuclei (the NNJL model), describing strong low-energy nuclear interactions, we compute the width of the energy level of the ground state of pionic deuterium. The theoretical value fits well the experimental data. Using the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , computed in the NNJL model, and the experimental values of the events of these reactions, detected by the SNO Collaboration, we compute the boron neutrino fluxes. The theoretical values agree well with the experimental data and the theoretical predictions within the Standard Solar Model by Bahcall. We argue the applicability of the constraints on the astrophysical factor for the solar proton burning, imposed by helioseismology, to the width of the energy level of the ground state of pionic deuterium. We show that the experimental data on the width satisfy these constraints. This testifies an indirect measurement of the recommended value of the astrophysical factor for the solar proton burning in terrestrial laboratories in terms of the width of the energy level of the ground state of pionic deuterium.

**PACS.** 11.10.Ef Lagrangian and Hamiltonian approach – 13.75.Gx Pion-baryon interactions – 36.10.-k Exotic atoms and molecules (containing mesons, muons, and other unusual particles) – 26.65.+t Solar neutrinos

## 1 Introduction

The Nuclear Nambu–Jona-Lasinio model of light nuclei [1–4] (the NNJL model) is an attempt to describe at the quantum field-theoretic level the deuteron as a bound  $np$  state [5]. As has been shown in [1–4], the NNJL model fits well the low-energy parameters of the deuteron, such as the binding energy, the dipole magnetic and electric quadrupole moments [1], the  $\Delta\Delta$  component [2] and

the asymptotic ratio  $D/S$  [4] of the wave function of the deuteron<sup>1</sup>.

The application of the NNJL model to the description of low-energy reactions of astrophysical interest [3] has allowed to compute: i) the cross-section for the neutron-proton radiative capture for thermal neutrons  $n+p \rightarrow d+\gamma$  in agreement with experimental data with an accuracy better than 3%; ii) the astrophysical factor for the solar proton burning  $p + p \rightarrow d + e^+ + \nu_e$ ,  $S_{pp}(0) = 4.08 \times 10^{-25}$  MeV b, agreeing well with the recommended value  $S_{pp}^{\text{SSM}}(0) = 4.00 \times 10^{-25}$  MeV b [8], accepted in the Standard Solar Model (SSM) by Bahcall [9,10]; iii) the

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<sup>1</sup> The asymptotic ratio  $D/S$  of the  $D$ -wave component to the  $S$ -wave component of the wave function of the deuteron in the ground state has been computed in the NNJL model in agreement with the results obtained by Ericson within the potential model approach [6] and the experimental value, which has been used by Kamionkowski and Bahcall [7] for the calculation of the astrophysical factor for the solar proton burning  $p + p \rightarrow d + e^+ + \nu_e$ .

astrophysical factor for the reaction  $p + e^- + p \rightarrow d + \nu_e$  in analytical agreement with the result obtained by Bahcall [11]; iv) the cross-sections for the reactor anti-neutrino disintegration of the deuteron  $\bar{\nu}_e + d \rightarrow n + n + e^+$  and  $\bar{\nu}_e + d \rightarrow p + n + \bar{\nu}_e$ , induced by the charged and neutral weak current, respectively, in agreement with the experimental data by the Reines Group [12].

In this paper we apply the NNJL model to the calculation of the width of the ground state of pionic deuterium. We show that the theoretical value agrees well with the experimental data. In the NNJL model the astrophysical factor for the solar proton burning and the width of the ground state of pionic deuterium are defined by the same matrix element, caused by the anomaly of the one-nucleon loop diagram. Due to this we suggest to apply the constraints on the astrophysical factor for the solar proton burning, imposed by helioseismology [13], to the width of the energy level of the ground state of pionic deuterium. We show that the available experimental data [14–16] on the width of the energy level of the ground state of pionic deuterium satisfy these constraints.

Remind that, according to the SSM [9,10], the astrophysical factor for the solar proton burning determines the temperature in the core of the Sun. Since the solar neutrino fluxes depend strongly on the solar core temperature [17], the precise knowledge of the temperature in the core of the Sun or, equivalently, the astrophysical factor for the solar proton burning, is very important for the correct definition of these fluxes [9,10].

As has been shown in [13], helioseismology imposes some constraints on the astrophysical factor  $S_{pp}(0)$  for the solar proton burning relative to the recommended value  $S_{pp}^{\text{SSM}}(0)$ . These constraints read

$$0.94 \leq \frac{S_{pp}(0)}{S_{pp}^{\text{SSM}}(0)} \leq 1.18. \quad (1.1)$$

Below we argue that through the NNJL model the same constraints can be applied to the width of the energy level of the ground state of pionic deuterium.

In order to make this assumption more credible and to give an additional confirmation that the NNJL model describes well strong low-energy interactions in nuclear reactions with the deuteron, we suggest to analyse the experimental data by the Sudbury Neutrino Observatory (SNO) [18,19] on the  ${}^8\text{B}$  solar neutrino flux measured through the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , caused by the charged and neutral weak current, respectively. For this aim, we use the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , computed within the NNJL model, and the experimental values of the rates of the events of these reactions, detected by the SNO Collaboration.

The paper is organized as follows. In sect. 2 we compute the  ${}^8\text{B}$  solar neutrino fluxes using the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , computed within the NNJL model and averaged over the  ${}^8\text{B}$  solar neutrino flux obtained by Bahcall *et al.* [20], and the experimental values of the rates of favourable events, detected by the SNO Collaboration. We show that the

${}^8\text{B}$  solar neutrino flux, computed through the reaction  $\nu_e + d \rightarrow p + n + \nu_e$  and caused by the neutral weak current, fits well the experimental data by the SNO Collaboration and the theoretical value, predicted within the SSM by Bahcall [10]. The obtained decrease of the  ${}^8\text{B}$  solar neutrino flux, computed through the cross-section for the reaction  $\nu_e + d \rightarrow p + p + e^-$  caused by the charged weak current, relative to that computed through the reaction  $\nu_e + d \rightarrow p + n + \nu_e$  can be explained by neutrino oscillations. This testifies that the NNJL model describes well strong low-energy interactions in low-energy nuclear reactions with the deuteron. In sect. 3 we compute the width of the energy level of the ground state of pionic deuterium within the NNJL model. We show that the theoretical value fits well the experimental data. In the conclusion we discuss the obtained results. We argue that the constraints on the astrophysical factor for the solar proton burning, imposed by the helioseismological data, can be applied to the width of the energy level of the ground state of pionic deuterium. We show that the experimental data on the width of the energy level of the ground state of pionic deuterium satisfy these constraints.

## 2 Theoretical analysis of SNO data on the solar neutrino disintegration of the deuteron

Recently [18] (see also [19]), the SNO Collaboration has published new experimental data on the  ${}^8\text{B}$  solar neutrino fluxes measured through the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , induced by the charged and neutral weak current,

$$\begin{aligned} \phi_{\text{CC}}^{\text{SNO}}({}^8\text{B}) &= (1.70 \pm 0.12) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \\ \phi_{\text{NC}}^{\text{SNO}}({}^8\text{B}) &= (4.90 \pm 0.38) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \end{aligned} \quad (2.1)$$

where the abbreviations CC and NC mean the *Charged* weak *Current* and the *Neutral* weak *Current*, respectively.

According to [21], the  ${}^8\text{B}$  solar neutrino fluxes measured through the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$  are defined by

$$\phi({}^8\text{B}) = 10^{-31} \frac{R}{\langle \sigma(E_{\nu_e}) \rangle_{\text{sB}}}, \quad (2.2)$$

where  $R$  is the experimentally measured rate of the favourable events,  $\langle \sigma(E_{\nu_e}) \rangle_{\text{sB}}$  is the theoretical cross-section for the reaction through which the  ${}^8\text{B}$  solar neutrino flux is measured. The cross-section is averaged over the  ${}^8\text{B}$  solar neutrino spectrum normalized to unity [20].

In our case the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$  are computed in the NNJL model [3] and averaged over the  ${}^8\text{B}$  solar neutrino spectrum obtained by Bahcall *et al.* [20]. Using the theoretical values for the cross-sections [3], the experimental values of the rates of favourable events, detected by the SNO Collaboration [18,19], we get

$$\begin{aligned} \phi({}^8\text{B})_{\text{CC}} &= (2.33 \pm 0.38) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}, \\ \phi({}^8\text{B})_{\text{NC}} &= (6.15 \pm 1.01) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}. \end{aligned} \quad (2.3)$$

It is seen that the cross-section for the reaction  $\nu_e + d \rightarrow p + n + \nu_e$ , computed within the NNJL model, fits well the experimental data by the SNO Collaboration on the  $^8\text{B}$  solar neutrino flux  $\phi_{\text{NC}}^{\text{SNO}}(^8\text{B}) = (4.90 \pm 0.38) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ .

We would like to emphasize that the  $^8\text{B}$  solar neutrino flux  $\phi(^8\text{B})_{\text{NC}} = (6.15 \pm 1.01) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$  agrees also well with the theoretical  $^8\text{B}$  solar neutrino flux, predicted within the SSM by Bahcall [10]:  $\phi^{\text{SSM}}(^8\text{B}) = (5.82 \pm 1.34) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ .

The cross-section for the reaction  $\nu_e + d \rightarrow p + p + e^-$ , induced by the charged weak current and computed within the NNJL model, leads to the theoretical prediction for the observed  $^8\text{B}$  solar neutrino flux, measured through the reaction  $\nu_e + d \rightarrow p + p + e^-$ , agreeing with the experimental value  $\phi_{\text{CC}}^{\text{SNO}}(^8\text{B}) = (1.70 \pm 0.12) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$  within two standard deviations but by a factor of 3 smaller compared with the  $^8\text{B}$  solar neutrino flux, measured through the reaction  $\nu_e + d \rightarrow p + n + \nu_e$ , induced by the neutral weak current.

According to the generally accepted point of view, such a distinction can be explained by neutrino oscillations [22, 23] (see also [10]). Remind that the cross-section for the reaction  $\nu_X + d \rightarrow p + n + \nu_X$  is practically insensitive to the neutrino flavour  $X = e, \mu$  or  $\tau$  [10, 23].

The obtained results testify that the NNJL model describes well strong low-energy interactions in low-energy nuclear reactions with the deuteron.

We would like to notice that the theoretical analysis of the cross-sections for the reactions of the neutrino disintegration of the deuteron  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , induced by charged and neutral weak currents, has a long history [24–26]. The calculation of the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$  has been carried out within other theoretical approaches such as i) the Potential Model Approach [24, 25] and ii) the Effective Field Theory approach [26]. The Effective Field Theory approach has been developed as a new tool for the description of low-energy phenomena in few-nucleon systems [27].

The theoretical cross-sections for the reactions of the neutrino disintegration of the deuteron  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , computed within the NNJL model in [3], are in reasonable agreement with the results, obtained within the Potential Model Approach [24, 25] and the Effective Field Theory approach [26]. The cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , averaged over the  $^8\text{B}$  solar neutrino spectrum, have been computed in [25]. The averaged values of the cross-sections, computed in the NNJL model and applied above to the analysis of the SNO data (2.1), are in qualitative agreement with those obtained in [25].

In this connection we would like to notice that for the calculation of the averaged values of the cross-sections for the reactions of the neutrino disintegration of the deuteron  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$  we have used the  $^8\text{B}$  solar neutrino spectrum [20], improved relative to those [28], which have been applied to the calculation of the average cross-sections in [25].

### 3 Width of the energy level of the ground state of pionic deuterium

According to Deser, Goldberger, Baumann and Thirring [29, 30] (see also [31–34]), the width of the energy level of the ground state of pionic deuterium is defined by the DGBT formula

$$\Gamma_{1s} = 4\alpha^3 m_\pi^2 \mathcal{I}m f_0^{\pi^- d}(0), \quad (3.1)$$

where  $\alpha = e^2 = 1/137.036$  is the fine-structure constant in Gaussian units and  $m_\pi = 140 \text{ MeV}$  is the pion mass,  $f_0^{\pi^- d}(0)$  is the  $S$ -wave amplitude of  $\pi^- d$  scattering near threshold.

For the analyses of the imaginary part of the amplitude  $f_0^{\pi^- d}(0)$ , it is sufficient to take into account the contribution of two processes,  $\pi^- d \rightarrow nn$  and  $\pi^- d \rightarrow nn\gamma$ , only. This defines the width (3.1) as follows:

$$\Gamma_{1s} = 4\alpha^3 m_\pi^2 (\mathcal{I}m f_0^{\pi^- d}(0)_{nn\gamma} + \mathcal{I}m f_0^{\pi^- d}(0)_{nn}) = \Gamma_{1s}^{(nn\gamma)} + \Gamma_{1s}^{(nn)}, \quad (3.2)$$

where  $\mathcal{I}m f_0^{\pi^- d}(0)_{nn\gamma}$  and  $\mathcal{I}m f_0^{\pi^- d}(0)_{nn}$  are the imaginary parts of the  $S$ -wave amplitudes of  $\pi^- d$  scattering near threshold saturated by the intermediate  $nn\gamma$  and  $nn$  states, and  $\Gamma_{1s}^{(nn\gamma)}$  and  $\Gamma_{1s}^{(nn)}$  are the partial widths of the decays  $A_{\pi d} \rightarrow nn\gamma$  and  $A_{\pi d} \rightarrow nn$ , respectively.

Following [31–34] (see also [35]), the  $S$ -wave amplitudes  $f_0^{\pi^- d}(0)_{nn\gamma}$  and  $f_0^{\pi^- d}(0)_{nn}$  can be defined by

$$\begin{aligned} f_0^{\pi^- d}(0)_{nn\gamma} &= \frac{1}{8\pi} \frac{1}{m_d + m_\pi} \frac{\alpha}{F_\pi^2} \frac{1}{2} \int \frac{d^3 p}{(2\pi)^2 2|\vec{p}|} \\ &\times \int \frac{d^3 k_1}{(2\pi)^3 2E_n(k_1)} \frac{d^3 k_2}{(2\pi)^3 2E_n(k_2)} \\ &\times (2\pi)^3 \delta^{(3)}(\vec{p} - \vec{k}_1 - \vec{k}_2) \\ &\times \frac{1}{E_n(k_1) + E_n(k_2) + |\vec{p}| - m_\pi - m_d - i0} \\ &\times \frac{1}{3} \sum_{\alpha_2=\pm 1/2} \sum_{\alpha_1=\pm 1/2} \sum_{\lambda_d=0,\pm 1} \sum_{\lambda=\pm 1} |e^{*\mu}(p, \lambda) \\ &\times \langle n(\vec{k}_1, \alpha_1) n(\vec{k}_2, \alpha_2) | J_{5\mu}^{1-i2}(0) | d(\vec{0}, \lambda_d) \rangle|^2 \end{aligned} \quad (3.3)$$

and

$$\begin{aligned} f_0^{\pi^- d}(0)_{nn} &= \frac{1}{128\pi} \frac{1}{m_d + m_\pi} \\ &\times \int \frac{d^3 k}{(2\pi)^3 E_n^2(k)} \frac{1}{E_n(k) - m_N - m_\pi/2 - i0} \\ &\times \frac{1}{3} \sum_{\alpha_2=\pm 1/2} \sum_{\alpha_1=\pm 1/2} \sum_{\lambda_d=0,\pm 1} |M(\pi^-(\vec{0})d(\vec{0}, \lambda_d) \rightarrow \\ &n(\vec{k}, \alpha_1)n(-\vec{k}, \alpha_2))|^2. \end{aligned} \quad (3.4)$$

In formula (3.2) the matrix element of the transition  $\pi^- d \rightarrow nn\gamma$  is given in the soft-pion limit [33, 35–40]. According to the Pauli principle [41], the  $nn$  pair in the reaction  $\pi^- d \rightarrow nn$ , where  $\pi^- d$  pair is in the  $S$ -wave state, can be only in the  $^3P_1$  state.

Computing the matrix element of the axial-vector current and the amplitude of the reaction  $\pi^-d \rightarrow nn$  in the NNJL model, for the partial widths of the decays  $A_{\pi d} \rightarrow nn\gamma$  and  $A_{\pi d} \rightarrow nn$  we obtain

$$\Gamma_{1s}^{(nn\gamma)} = m_\pi^2 m_N^2 g_V^2 C_{NN}^2 \frac{3\alpha^4}{64\pi^7} \frac{g_A^2}{F_\pi^2} \times \int_0^\infty \frac{dk k^2 F_d^2(k^2)}{(1 - \frac{1}{2} r_{nn} a_{nn} k^2)^2 + a_{nn}^2 k^2}. \quad (3.5)$$

and

$$\Gamma_{1s}^{(nn)} = \alpha^3 C_{NN}^2 \frac{m_\pi^4}{F_\pi^2} \frac{3g_A^2 g_V^2}{256\pi^7} \times k_0^3 F_d^2(k_0^2) |f_{\pi^-d}^{(nn;^3P_1)}(k_0)|^2, \quad (3.6)$$

where  $g_A = 1.267$  is the axial-vector coupling constant,  $F_\pi = 92.4$  MeV is the leptonic constant of charged pions,  $g_V = 11.3$  and  $C_{NN} = 3.27 \times 10^{-3}$  MeV $^{-2}$  are the coupling constants of the NNJL model,  $F_d(k^2) = 1/(1 + r_d^2 k^2)$  is the form factor of the deuteron, proportional to the wave function of the ground state of the deuteron in the momentum representation,  $r_d = 4.32$  fm =  $3.07$  m $_\pi^{-1}$  is the deuteron radius [42],  $a_{nn}$  and  $r_{nn}$  are the  $S$ -wave scattering length of the  $nn$  scattering in the  $^1S_0$  state. For numerical calculation we use  $a_{nn} = -23.75$  fm =  $-16.85$  m $_\pi^{-1}$  and  $r_{nn} = 2.75$  fm =  $1.95$  m $_\pi^{-1}$  [3]. Then, the relative momentum  $k_0$  of the  $nn$  pair at threshold of the reaction  $\pi^-d \rightarrow nn$  in the center-of-mass frame is equal to  $k_0 = \sqrt{m_\pi m_N} = 362$  MeV. The amplitude  $f_{\pi^-d}^{(nn;^3P_1)}(k_0)$  describes the final-state interaction of the  $nn$  pair in the  $^3P_1$  state near threshold of the reaction  $\pi^-d \rightarrow nn$ . Following [34], we compute  $|f_{\pi^-d}^{(nn;^3P_1)}(k_0)| = 0.7$ .

In formula (3.5) the integral over  $k$  amounts to  $0.016/r_d^3$ . The theoretical values of the partial widths of the decays  $A_{\pi d} \rightarrow nn\gamma$  and  $A_{\pi d} \rightarrow nn$  read

$$\begin{aligned} \Gamma_{1s}^{(nn\gamma)} &= (0.30 \pm 0.04) \text{ eV}, \\ \Gamma_{1s}^{(nn)} &= (0.85 \pm 0.11) \text{ eV}. \end{aligned} \quad (3.7)$$

According to (3.2), for the total width of the energy level of the ground state of pionic deuterium we get

$$\Gamma_{1s} = (1.15 \pm 0.12) \text{ eV}. \quad (3.8)$$

Our theoretical value of the width  $\Gamma_{1s} = (1.15 \pm 0.12)$  eV agrees well with the experimental data

$$\Gamma_{1s}^{\text{exp}} = \begin{cases} (1.02 \pm 0.21) \text{ eV} & [14, 15], \\ (1.19 \pm 0.11) \text{ eV} & [16]. \end{cases} \quad (3.9)$$

The partial widths  $\Gamma_{1s}^{(nn\gamma)}$  and  $\Gamma_{1s}^{(nn)}$  can be also related by the parameter  $D$ :

$$D = \frac{\sigma(\pi^-d \rightarrow nn)}{\sigma(\pi^-d \rightarrow nn\gamma)} = \frac{\Gamma_{1s}^{(nn)}}{\Gamma_{1s}^{(nn\gamma)}} = 2.83 \pm 0.04, \quad (3.10)$$

measured experimentally at threshold of the reactions  $\pi^-d \rightarrow nn$  and  $\pi^-d \rightarrow nn\gamma$  [43]. Using the theoretical values of the partial widths (3.7) we compute the parameter  $D$ :  $D = 2.83 \pm 0.50$ . It agrees well with the experimental data (3.10).

## 4 Conclusion

We have applied the NNJL model to the calculation of the width of the energy level of the ground state of pionic deuterium. Without introduction of new input parameters, we have computed the value of the width of the energy level of the ground state of pionic deuterium  $\Gamma_{1s} = (1.15 \pm 0.12)$  eV in complete agreement with the experimental data (3.9).

Remind that the NNJL model has been invented for the quantum field-theoretic description of the deuteron as a bound  $np$  state and low-energy nuclear reactions with the deuteron of the astrophysical interest such as the solar proton burning and so on. However, as has turned out the NNJL model can be also applied to the calculation of the width of the energy level of the ground state of pionic deuterium, since the amplitudes of the solar proton burning  $p + p \rightarrow d + e^+ + \nu_e$ , the  $pep$  reaction  $p + e^- + p \rightarrow d + \nu_e$ , the neutrino disintegration of the deuteron  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$  and the reactions  $\pi^- + d \rightarrow n + n + \gamma$  and  $\pi^- + d \rightarrow n + n$  near threshold of the  $\pi^-d$  pair are defined by the anomaly of the same one-nucleon loop diagram [3].

Since in the SSM the astrophysical factor of the solar proton burning is related to the temperature of the solar core, the helioseismological data become sensitive to the value of the astrophysical factor for the solar proton burning. The constraints on the value of the astrophysical factor for the solar proton burning, coming from the helioseismological data on the values of sound speed and density inside the Sun, have been found by Degl'Innocenti, Fiorentini and Ricci [13].

Since the NNJL model fits well the recommended value of the astrophysical factor for the solar proton burning and the experimental data on the width of the energy level of the ground state of pionic deuterium, one can imagine that the constraints on the astrophysical factor for the solar proton burning, imposed by helioseismology (1.1), can be also valid for the width of the energy level of the ground state of pionic deuterium. This yields

$$(1.08 \pm 0.11) \text{ eV} \leq \Gamma_{1s} \leq (1.36 \pm 0.14) \text{ eV}. \quad (4.1)$$

It is seen that the experimental data (3.9) satisfy well the constraints (4.1).

Moreover, since the astrophysical factor for the solar proton burning,  $S_{pp}(0) = 4.08 \times 10^{-25}$  MeV b, computed within the NNJL model, fits the recommended value  $S_{pp}^{\text{SSM}}(0) = 4.00 \times 10^{-25}$  MeV b with an accuracy about 2%, our prediction for the width of the energy level of the ground state of pionic deuterium, agreeing with the experimental data with an accuracy about 3%, can be valued as an indirect measurement of the recommended value of the astrophysical factor  $S_{pp}^{\text{SSM}}(0) = 4.00 \times 10^{-25}$  MeV b in terrestrial laboratories in terms of the width of the energy level of the ground state of pionic deuterium.

For the confirmation of the applicability of the NNJL model to the description of strong low-energy interactions with the deuteron and the results obtained above, we have analysed the experimental data on the  $^8\text{B}$  solar neutrino flux measured by the SNO Collaborations. Using

the cross-sections for the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , computed within the NNJL model and averaged over the  $^8\text{B}$  solar neutrino spectrum by Bahcall *et al.* [20], and the experimental values of the rates of the events of the reactions  $\nu_e + d \rightarrow p + p + e^-$  and  $\nu_e + d \rightarrow p + n + \nu_e$ , detected by the SNO Collaboration, we have computed the  $^8\text{B}$  solar neutrino fluxes.

The computed value  $\phi(^8\text{B})_{\text{NC}} = (6.15 \pm 1.01) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$  of the  $^8\text{B}$  solar neutrino flux, measured through the reaction  $\nu_e + d \rightarrow p + n + \nu_e$ , agrees well with the experimental data and the theoretical value of the  $^8\text{B}$  solar neutrino flux  $\phi^{\text{SSM}}(^8\text{B}) = (5.82 \pm 1.34) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ , predicted within the SSM by Bahcall [10].

In turn, the computed value  $\phi(^8\text{B})_{\text{CC}} = (2.33 \pm 0.38) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$  of the  $^8\text{B}$  solar neutrino flux, measured through the reaction  $\nu_e + d \rightarrow p + p + e^-$ , agrees with the experimental data within two standard deviations but differs by a factor of 3 from the  $^8\text{B}$  neutrino flux  $\phi(^8\text{B})_{\text{NC}} = (6.15 \pm 1.01) \times 10^6 \text{ cm}^{-2} \text{ s}^{-1}$ . However, nowadays there is a consensus [10,23] that such a distinction can be explained by solar neutrino oscillations.

Such an agreement of the computed  $^8\text{B}$  solar neutrino fluxes with the experimental data by the SNO Collaboration and the theoretical predictions of the SSM by Bahcall [10] testify that the NNJL model describes well strong low-energy interactions in low-energy nuclear reactions with the deuteron.

This makes also credible our assumption concerning the applicability of the constraints on the solar proton burning, coming from helioseismology, to the width of the ground state of pionic deuterium and vice versa.

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